Multiplying and Dividing Integers

Some Notes on Notation

You have been writing integers with raised signs to avoid confusion with the symbols for addition and subtraction. However, most computer software and most writing in mathematics do not use raised signs.

Positive numbers are usually written without a sign.

$$^+3 = 3 \text{ and } ^+7.5 = 7.5$$

Negative numbers are usually written with a dash like a subtraction sign.

$$^-3 = -3 \text{ and } ^-7.5 = -7.5$$

From now on, we will use this notation to indicate a negative number.

This can be confusing if you don't read carefully. Parentheses can help.

$$^-5 - ^-8 = -5 - -8 = -5 - (-8)$$

The subtraction symbol also indicates the opposite of a number. For example, -8 represents the opposite of 8. The expression $-(-8)$ represents the opposite of -8.

$$-(-8) = 8$$

For multiplication, you can use a raised dot symbol.

$$3 \times 5 = 3 \cdot 5$$

In this investigation, you will use time, distance, speed, and direction to think about multiplication and division of integers. You will also look at number patterns and develop algorithms for multiplying and dividing these numbers.

Did You Know?

Michael Johnson set a world record by running 400 meters in 43.18 seconds at the world track championships in 1999. Florence Griffith Joyner set an Olympic record when she ran 100 meters in 10.62 seconds in 1988.

*How long would it take each runner to run 1,000 meters at his or her record speed?*

Go Online
PHSchool.com
For: Information about track
Web Code: ane-9031

3.1 Introducing Multiplication of Integers

The math department at Everett Middle School sponsors a contest called the Number Relay. A number line measured in meters is drawn on the school field. Each team has five runners. Runners 1, 3, and 5 stand at the −50 meter line. Runners 2 and 4 stand at the 50 meter line.
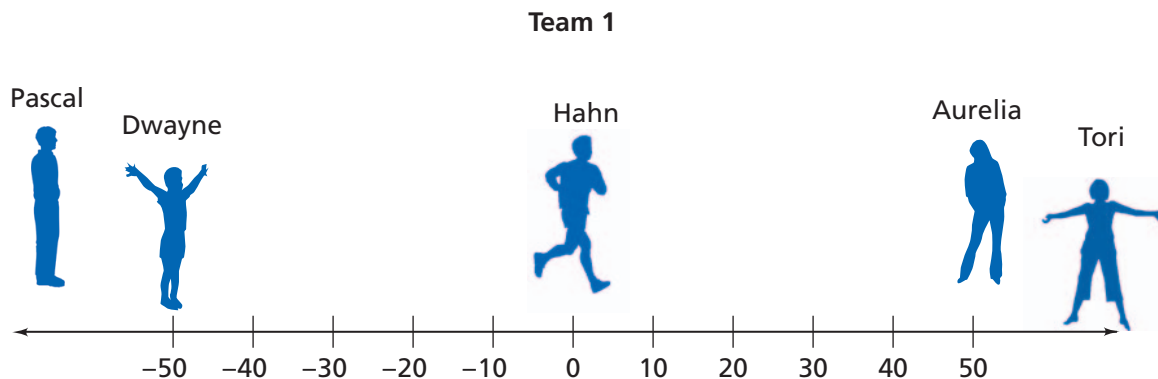
Team 1

For Team 1:

- Hahn starts and runs from −50 to 50. He tags Aurelia.
- Aurelia runs back from 50 to −50. She tags Dwayne.
- Dwayne runs from −50 to 50. He tags Tori.
- Tori runs from 50 to −50. She tags Pascal.
- Pascal runs from −50 to the finish line at position 0.

The team whose final runner reaches the 0 point first wins.

A. Write number sentences that express your answers to these questions. Use positive numbers for running speeds to the right and negative numbers for running speeds to the left. Use positive numbers for time in the future and negative numbers for time in the past. Each runner runs at a constant speed.

1. Hahn passes the 0 point running 5 meters per second to the right. Where is he 10 seconds later?

2. Dwayne passes the 0 point running 4 meters per second to the right. Where is he 5 seconds later?

3. Aurelia passes the 0 point running to the left at 6 meters per second. Where is she 8 seconds later?

4. Pascal passes the 0 point running to the right at 3 meters per second. Where was he 6 seconds earlier?

5. Tori passes the 0 point running to the left at 5 meters per second. Where was she 7 seconds earlier?

B. 1. Find the products in each group below.

Group 1	Group 2	Group 3
4×3	$4 \times (-3)$	$-4 \times (-3)$
5.1×1	-1.5×2	$-7 \times (-4)$
3×4.5	$10 \times (-11)$	$-5.2 \times (-1)$

2. Describe what the examples in each group have in common.

3. Use your answer to part (2) to write two problems for each group.

4. Describe an algorithm for multiplying rational numbers.

5. Use your strategy to multiply these rational numbers.
   a. $-1\frac{1}{2} \times \frac{3}{4}$ **b.** $-\frac{1}{2} \times \left(-\frac{3}{4}\right)$ **c.** $2\frac{1}{2} \times \left(-\frac{3}{4}\right)$

6. Is multiplication commutative? Does the order of factors matter? For example, are these multiplication sentences correct?

$$2 \cdot 3 \overset{?}{=} 3 \cdot 2$$
$$-2 \times (-3) \overset{?}{=} -3 \times (-2)$$
$$-2 \times 3 \overset{?}{=} 3 \times (-2)$$

ACE **Homework starts on page 50.**

3.2 Multiplication Patterns

After studying the relay race problem, some students started playing with number patterns to see whether what they found in the relay race made sense. Study the equations below. Look for patterns.

$$5 \times 5 = 25$$
$$5 \times 4 = 20$$
$$5 \times 3 = 15$$
$$5 \times 2 = 10$$
$$5 \times 1 = 5$$
$$5 \times 0 = 0$$

Problem 3.2 Multiplication Patterns

A. 1. How do the products change as the numbers multiplied by 5 get smaller?

   2. Predict $5 \times (-1), 5 \times (-2)$, and $5 \times (-3)$. Explain your reasoning.

   3. Write the next four equations in the pattern.

B. 1. Complete the equations below.

$$(-4) \times 5 = \blacksquare$$
$$(-4) \times 4 = \blacksquare$$
$$(-4) \times 3 = \blacksquare$$
$$(-4) \times 2 = \blacksquare$$
$$(-4) \times 1 = \blacksquare$$
$$(-4) \times 0 = \blacksquare$$

   2. How do the products change as the numbers multiplied by -4 get smaller?

   3. Predict $-4 \times (-1)$. Explain.

   4. Write the next four equations in the pattern.

C. 1. Find each value.

   a. $7 \times (-8) \times (-3)$

   b. $-12 \times (-5) \times (-4)$

   c. $\frac{1}{2} \times \left(-\frac{2}{3}\right) \times 3$

   2. How do the patterns you found in this problem compare to the algorithm from Problem 3.1?

ACE Homework starts on page 50.
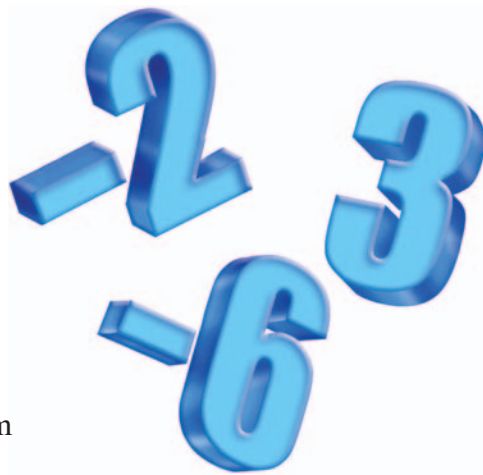
Introducing Division of Integers

You know there is a relationship between addition and subtraction facts. A similar relationship exists between multiplication and division. For any multiplication fact, we can write another multiplication fact and two different related division facts. Here are three examples.

$\left(\text{Remember that you can write } 15 \div 3 \text{ as a fraction, } \frac{15}{3}.\right)$

Example 1	Example 2	Example 3
$5 \times 3 = 15$	$6 \times (-3) = -18$	$4.5 \times (-2) = -9$
$3 \times 5 = 15$	$-3 \times 6 = -18$	$-2 \times 4.5 = -9$
$15 \div 3 = 5 \text{ or } \frac{15}{3} = 5$	$-18 \div (-3) = 6 \text{ or } \frac{-18}{-3} = 6$	$-9 \div (-2) = 4.5 \text{ or } \frac{-9}{-2} = 4.5$
$15 \div 5 = 3 \text{ or } \frac{15}{5} = 3$	$-18 \div 6 = -3 \text{ or } \frac{-18}{6} = -3$	$-9 \div 4.5 = -2 \text{ or } \frac{-9}{-4.5} = -2$

Getting Ready for Problem 3.3

- What patterns do you see in Examples 1–3?
- Write a fact family for $-2 \times 3 = -6$.
- How can you use what you know about the relationship between multiplication and division facts to help you solve these problems?

  1. $8 \times \blacksquare = -48$

  2. $\blacksquare \times (-9) = 108$

  3. $6 \times (-13) = \blacksquare$

You can use this relationship and your ideas from the Number Relay questions to develop algorithms for dividing integers.

Problem 3.3 Introducing Division of Integers

A. Recall the Number Relay from Problem 3.1. Write division sentences that express your answers to the questions below.

1. Dwayne goes from 0 to 15 in 5 seconds. At what rate (distance per second) does he run?

2. Aurelia reaches -12 only 3 seconds after passing 0. At what rate does she run to the left?

3. Pascal passes 0 running to the right at a rate of 5 meters per second. When did he leave the point -50? When did he leave the point -24?

4. Tori wants to reach the point -40 running to the left at 8 meters per second. How long will it take her from the time she passes 0?

B. 1. Find the quotients in each group below.

Group 1	Group 2	Group 3
$12 \div 3$	$39 \div (-3)$	$-45 \div (-3)$
$51 \div 5$	$-15 \div 4$	$-4.8 \div (-4)$
$4.5 \div 9$	$10 \div (-5)$	$-72 \div (-12)$

2. Describe what the examples in each group have in common.

3. Use your answer to part (2) to write two problems for each group.

4. Describe an algorithm for dividing rational numbers.

5. Use your strategy to divide these rational numbers.

a. $-1\frac{1}{2} \div \frac{3}{4}$ **b.** $-\frac{1}{2} \div \left(-\frac{3}{4}\right)$ **c.** $2\frac{1}{2} \div \left(-\frac{3}{4}\right)$

6. Is division commutative? Does $-2 \div 3 = 3 \div (-2)$? Give two other examples to support your answer.

ACE Homework starts on page 50.

3.4 Playing the Integer Product Game

You have developed algorithms for multiplying and dividing integers. You will need them to play the Integer Product Game.

The game board consists of a list of factors and a grid of products. To play, you need a game board, two paper clips, and colored markers or chips.

Integer Product Game Rules

1. Player A puts a paper clip on a number in the factor list.
2. Player B puts the other paper clip on any number in the factor list, including the number chosen by Player A. Player B then marks the product of the two factors on the product grid.
3. Player A moves *either one* of the paper clips to another number. He or she then marks the new product with a different color than Player B.
4. Each player takes turns moving a paper clip and marking a product. A product can only be marked by one player.
5. The winner is the first player to mark four squares in a row (up and down, across, or diagonally).

Integer Product Game Board

−36	−30	−25	−24	−20	−18
−16	−15	−12	−10	−9	−8
−6	−5	−4	−3	−2	−1
1	2	3	4	5	6
8	9	10	12	15	16
18	20	24	25	30	36

Factors:

−6 −5 −4 −3 −2 −1 1 2 3 4 5 6

Problem 3.4 Multiplying Integers

Play the Integer Product Game with positive and negative factors. Look for strategies for picking the factors and products.

A. What strategies did you find useful in playing the game? Explain.

B. What pair(s) of numbers from the factor list will give each product?

   1. 5   **2.** -12 **3.** 12   **4.** -25

C. Your opponent puts a paper clip on -4. List five products that you can form. Tell where you need to put your paper clip in each case.

D. Describe the moves to make in each case.

   1. The paper clips are on -5 and -2. You want a product of -15.

   2. The paper clips are on -3 and -2. You want a product of -6.

   3. Your opponent will win with 24. What numbers should you avoid with your paper clip moves?

active math online

ACE Homework starts on page 50.